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Influence of symmetry breaking on the fluctuation properties of spectra

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Abstract
We study the effect of gradual symmetry breaking in a non-integrable system on the level fluctuation statistics. We consider the case when the symmetry is represented by a quantum number that takes one of two possible values, so that the unperturbed system has a spectrum composed of two independent sequences. When symmetry-breaking perturbation is represented by a random matrix with an adjustable strength, the shape of the spectrum monotonically evolves towards the Wigner distribution as the strength parameter increases. This contradicts the observed behaviour of the acoustic resonance spectra in quartz blocks during the breaking of a point-group symmetry, which are changed in the beginning towards the Poisson statistics and then revert to the GOE statistics. This behaviour is explained by assuming that the symmetry-breaking perturbation removes the degeneracy due to symmetry for a limited number of levels, thus creating a third chaotic sequence. As symmetry breaking increases, the new sequence grows at the expense of the initial pair until it overwhelms the whole spectrum when the symmetry completely disappears. The calculated spacing distribution and spectral rigidity are able to describe the evolution of the acoustic resonance spectra.

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1. Introduction

The statistical theory of spectra [1] provides an appropriate method for examining the symmetry properties of quantum systems. Level statistics such as the nearest-neighbour spacing (NNS) distribution and the spectral rigidity Δ_3 are defined for a pure sequence of levels that have the same quantum numbers. Usually it is taken for granted that in a classically integrable system, which has as many integrals of motion (or quantum numbers) as the number of degrees of freedom, the levels are uncorrelated and so have a Poissonian NNS distribution. The pure level sequence in a time-reversal-invariant quantum system whose classical counterpart is chaotic, is successfully represented by a Gaussian orthogonal ensemble (GOE) of random matrices.

These well-established conjectures are applicable to completely integrable or chaotic systems. There are, however, intermediate situations. One is of mixed systems, where in some parts of the corresponding phase space the motion is regular and in other parts it is chaotic [2–4]. Other types of systems that exhibit intermediate statistics are those with singularities that are integrable in the absence of these singularities (see, e.g., [5–7]).

The statistical theory of spectra has also been extensively used in the investigation of symmetry breaking because the destruction of a quantum number has a dramatic effect on spectral fluctuations. The studies of isospin mixing in light nuclei are impressive [8]. Shriner *et al* [9] measured a 'complete' spectrum of low-lying states of ^{26}Al involving states with isospins $T = 0$ and 1 throughout the energy range covered by the data. These data offered a testing ground for studying the influence of isospin-symmetry breaking on the fluctuation properties of energy spectra (see, e.g., [10–13]). However, the limited data available in ^{26}Al (142 levels) precluded a definite conclusion. In a recent experiment with monocrystalline quartz blocks, Ellegaard *et al* [14] measured about 1400 well-resolved acoustic resonances. The properties of quartz allowed these authors to measure the gradual break-up of a point-group symmetry, which is statistically fully equivalent to the breaking of a quantum number such as isospin. The spectra are obtained by externally tuning the symmetry breaking [14], allowing a statistically significant investigation of the whole transition.

In the present paper we consider the effect of gradual symmetry breaking on the fluctuation properties of energy levels. When the symmetry is full the spectrum is divided into independent sequences, each corresponding to one of the eigenvalues of the symmetry operator. We consider two possible scenarios for breaking the symmetry. The first is provided by the Guhr–Weidenmüller model [11], which assumes that the matrix elements between states of different symmetry eigenvalues are random numbers of equal variances. In the second scenario, the symmetry-breaking interaction mixes a limited number of the degenerate eigenstates that belong to different symmetry representations, and this number grows as we increase symmetry breaking. We are thus creating a new sequence of levels corresponding to states with no symmetry at the expense of the initial sequences. Consequently, as the fractional density of the no-symmetry sequence increases, the Δ_3 statistic of the total spectrum first increases, moving in the direction of the prediction of the Poisson statistics until the densities of all the sequences become equal, and then decreases to reach the value given by the GOE asymptotically. Section 3 shows that the behaviour described by the second scenario is indeed present in the data of Ellegaard *et al* [14]. The summary and conclusion of this study are given in section 4.

2. Symmetry breaking in a chaotic system

We consider a chaotic system that conserves a given symmetry. In such a system, the Hamiltonian assumes a block-diagonal form [1]:

$$H_S = \mathrm{diag}(H_1, H_2, \ldots, H_n) \tag{1}$$

where each sub-block corresponds to states of a certain quantum number. Thus, when the symmetry is conserved, the levels are divided into a number of 'pure' sequences, each described by a pure GOE. However, the statistical properties of the total spectrum that combines these sequences are no longer of the GOE type. In the limit of infinite number of sequences combined, the spectrum obeys Poisson statistics. For finite number of sequences, the fluctuation properties are intermediate between the Poisson and GOE statistics. In the following, we shall restrict our consideration to the case when the symmetry under investigation is represented by an operator that has two possible eigenvalues, such as parity, or isospins 0 and 1 as considered in [9–13].

The NNS distribution of a spectrum resulting from a random superposition of n independent sequences is calculated, e.g. by Berry and Robnik [2] and in Mehta's book [15]. If the level density of the ith sequence is ρ_i, and if the NNS distribution of levels of this sequence is $P_i(x_i)$, where $x_i = f_i s$, $f_i = \rho_i/\Sigma\rho_i$, and s is the NNS normalized to a unit mean, then the NNS distribution of the mixed sequence is given by

$$P(s) = E(s)\left\{\sum_i f_i^2 \frac{P_i\,(f_i s)}{E_i\,(f_i s)} + \left[\sum_i f_i \frac{1-W_i(f_i s)}{E_i(f_i s)}\right]^2 - \sum_i \left[f_i \frac{1-W_i(f_i s)}{E_i(f_i s)}\right]^2\right\} \tag{2}$$

where $E(s) = \prod_{i=1}^n E_i(f_i s)$, $E_i(x_i) = \int_{x_i}^\infty [1 - W_i\,(x)]\,\mathrm{d}x$, and $W_i(x_i) = \int_0^{x_i} P_i(x)\,\mathrm{d}x$. In particular, if all the n individual sequences have the same level densities, so that $f_i = 1/n$, and if the NNS distribution in each is a Wigner distribution

$$P_i(x_i) = \frac{\pi}{2} x_i\,\mathrm{e}^{-\frac{\pi}{4}x_i^2} \tag{3}$$

which is a good approximation for the NNS distribution of a GOE, then (2) becomes [16]

$$P_n(s) = \frac{1}{n}\left[\mathrm{erfc}\left(\frac{s\sqrt{\pi}}{2n}\right)\right]^n Q_n(s)\left[\frac{\pi}{2n}s + (n-1)Q_n(s)\right] \tag{4}$$

where $Q_n(s) = \exp(-\pi^2 s^2/4n^2)/\mathrm{erfc}(s\sqrt{\pi}/2n)$ and $\mathrm{erfc}(x)$ is the complementary error function. This distribution has a shape intermediate between those of the Wigner and Poisson distributions, but quite different from the Brody distribution [17],

$$P_{\mathrm{B}}\,(s) = c_\beta s^\beta \exp\left(-\frac{c_\beta}{\beta+1}s^{\beta+1}\right) \qquad \text{with} \qquad c_\beta = \frac{\Gamma^{\beta+1}\,(1/\,(\beta+1))}{\beta+1} \tag{5}$$

which accurately reproduces the spectra of many two-dimensional systems with mixed regular-chaotic dynamics. As β varies from 0 to 1, this distribution changes from the Poissonian form to the GOE.

The NNS distribution contains information about the spectrum in a short range, not exceeding a maximum of three mean level spacings. Long-range information is provided by various higher order correlation functions [1, 15]. The most popular among these is the spectral rigidity Δ_3 of Dyson and Mehta [18]. It measures the mean square deviation of the integrated density of states from a straight line in an interval of length L, averaged over the starting point of the interval. Semiclassical arguments [19] show that, for a generic integrable system whose spectrum satisfies the Poisson statistics

$$\Delta_{3,\mathrm{Poisson}}(L) = \frac{L}{15} \qquad \text{for} \quad L \ll L_{\max} \tag{6}$$

where $L_{\max} = 2\pi\hbar/DT_{\min}$, D is the mean level spacing and $T_{\min}$ is the period of the shortest classical orbit. For a time-reversal-invariant chaotic system the semiclassical theory gives the following asymptotic expression

$$\Delta_{3,\mathrm{GOE}}(L) = \frac{1}{\pi^2}\ln L - 0.007 \qquad \text{for} \quad 1 \ll L \ll L_{\max}. \tag{7}$$

In all cases Δ_3 saturates to a non-universal value at $L \sim L_{\max}$. The random matrix theory obtains an analytical expression for $\Delta_{3,\mathrm{GOE}}(L)$ that involves double integration [1, 15]. We find it more suitable to parametrize this expression for a GOE, guided by equation (7), in the form

$$\Delta_{3,\mathrm{GOE}}(L) = \frac{1}{\pi^2}(1-\mathrm{e}^{-aL})[\ln(L+b)+c] \tag{8}$$

with $a = 8.24$, $b = 0.944$ and $c = -0.0672$ being the best fit parameters for values of L in the range $0 \leqslant L \leqslant 50$. Seligman and Verbaarschot [20] suggested the following expression for the spectral rigidity of a spectrum resulting from a random superposition of n independent sequences with fractional level densities f_i

$$\Delta_3(L) = \sum_{i=1}^{n} [\Delta_{3,i}(f_i L)] \tag{9}$$

where $\Delta_{3,i}(L)$ is the spectral rigidity for the ith sequence. This proposal is not surprising because $\Delta_3(L)$ is essentially a variance.

We now consider symmetry breaking in the chaotic system using the Guhr–Weidenmüller model [11]. The model suggests that the effect of the symmetry-breaking perturbation on all the unperturbed states is equal in the average. This suggests defining the Hamiltonian in the form

$$H = \begin{bmatrix} H_0 & 0 \\ 0 & H_0 \end{bmatrix} + \alpha \begin{bmatrix} 0 & W \\ W^T & 0 \end{bmatrix} \tag{10}$$

where H_0 represents a GOE and W is a random matrix whose elements have Gaussian distributions with zero means and variances equal to those of the non-diagonal elements of H_0, while W^T is its transpose. When $\alpha = 0$, the symmetry is conserved and the Hamiltonian has the block-diagonal form (1) with $n = 2$. The spectrum consists of an independent superposition of two level sequences having equal density. The NNS distribution $P(s)$ is given by equation (4) and, in particular, $P(0) = 1/2$. The spectral rigidity is given by (9) with $f_1 = f_2 = 1/2$. When we allow α to increase, the system gradually acquires the behaviour of a single GOE. Leitner [21] has shown this analytically using an approximation which is strictly valid when α is small. To show how the transition from 2-GOE statistics to that of a single GOE proceeds, we considered an ensemble of ten 200×200 Hamiltonian matrices of the form (10) for a fixed value of the perturbation strength α. We numerically diagonalized these matrices. We obtained the NNS distribution and spectral rigidity for each of the resulting spectra and measured their deviation from the GOE statistics by comparing with the Brody distribution (5) for the NNS distribution and with the Seligman–Verbaarschot formula (9) for the Δ_3 statistic where $\Delta_{3,1}$ is given by the Poissonian statistic (6) and $\Delta_{3,2}$ by that of a GOE as approximated by (8):

$$\Delta_{3,\mathrm{SV}}(L) = q_{\mathrm{SV}} \frac{L}{15} + \Delta_{3,\mathrm{GOE}}[(1 - q_{\mathrm{SV}})L]. \tag{11}$$

We then repeated the procedure for other value of α. The best fit values of the tuning parameters β and q_{SV} obtained for $P(s)$ and $\Delta_3(L)$ corresponding to different values of the perturbation strength α are shown in figure 1. The monotonic nature of the curves clearly shows that the transition from symmetry to no-symmetry in the Guhr–Weidenmüller model is gradual and one-directional.

The opposite extreme of the Guhr–Weidenmüller model is the case when the eigenfunctions become either very weakly or very strongly perturbed. The number of eigenstates in each category depends on the degree of symmetry breaking. We shall consider the limiting case when N_1 and N_2 of the states are so weakly affected by the perturbation that the eigenfunctions can in principle be characterized by one of the two symmetry eigenvalues. These will be the states for which the splitting of two degenerate levels belonging to different symmetry representation can be neglected. All the remaining N_3 states will be assumed to be

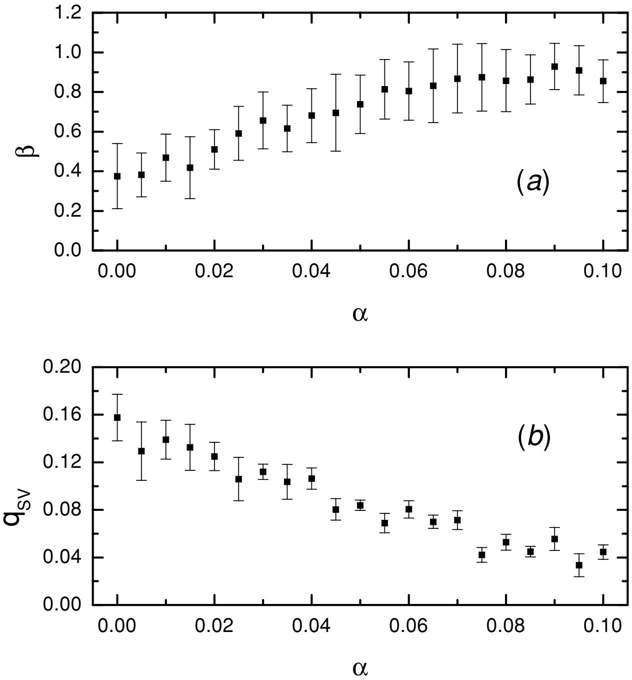

Figure 1. The monotonic dependence of (*a*) the tuning parameter β in the Brody distribution (equation (5)) for the NNS distributions, $P_{\mathrm{B}}(s)$, and (*b*) the parameter q_{SV} in the Seligman–Verbaarschot formula (equation (11)) for the spectral rigidity $\Delta_3(L)$ that interpolate between the Poisson and GOE statistics, obtained by a numerical experiment on a chaotic system undergoing a gradual breaking of a symmetry according to the Guhr–Weidenmüller model [11].

strongly mixed by the symmetry-breaking perturbation. In this case the Hamiltonian cannot be represented by the form (10), and may rather be expressed as

$$H = \begin{bmatrix} H_1 & 0 & 0 \\ 0 & H_3 & 0 \\ 0 & 0 & H_2 \end{bmatrix} \tag{12}$$

where H_i is a matrix of dimension N_i that satisfies the GOE statisics. We shall again assume equal number of states for each symmetry states, $N_1 = N_2 = N/2$. The corresponding levels will again form three independent level sequences. Two of them represent the weakly unperturbed states, and have fractional level densities $f_1 = f_2 = f/2$, where $f = N/(N + N_3)$. The levels of the strongly perturbed eigenstates will form the third sequence with density $f_3 = 1 - f$ which will grow at the expense of the other two as the symmetry-breaking perturbation increases. The NNS distribution will then be given, using (2) and (3), by

$$\begin{aligned} P(s) = {} & \frac{\pi}{8} f^3 s\, \mathrm{e}^{-\pi f^2 s^2/16}\, \mathrm{erfc}\left(\frac{\sqrt{\pi}}{4} f s\right) \mathrm{erfc}\left[\frac{\sqrt{\pi}}{2}(1-f)\, s\right] + \frac{1}{2} f^2\, \mathrm{e}^{-\pi f^2 s^2/8} \\ & \times \mathrm{erfc}\left[\frac{\sqrt{\pi}}{2}(1-f)\, s\right] + \frac{\pi}{2}(1-f)^3\, s\, \mathrm{e}^{-\pi(1-f)^2 s^2/4}\left[\mathrm{erfc}\left(\frac{\sqrt{\pi}}{4} f s\right)\right]^2 \\ & + 2f(1-f)\, \mathrm{e}^{-\pi\left(5f^2-8f+4\right)s^2/16} \mathrm{erfc}\left(\frac{\sqrt{\pi}}{4} f s\right) \end{aligned} \tag{13}$$

for spacings exceeding a certain lower limit. If we ignore the latter condition and extrapolate $P(s)$ into the origin, we obtain

$$P(0) = f\left(2 - \tfrac{3}{2}f\right) \tag{14}$$

which increases with decreasing f from a value of $1/2$ at $f = 1$, reaches a maximum value of $2/3$ at $f = 2/3$ and then deceases monotonically until it vanishes at $f = 0$. Equations (13) and (14) show that the evolution of the shape of the NNS distributions with increasing symmetry breaking and decreasing f, is not a straightforward transition from the 2-GOE behaviour to that of a GOE. Indeed, by decreasing the value of f we mean removing some of the levels from the initial two symmetry-invariant sequences to form the third sequence that represents the states without symmetry. The distribution will look more regular when we decrease f until the level densities of the three sequences become equal. If we further increase the symmetry breaking, the initial sequences become thinner as more levels move to the new chaotic group until the whole spectrum forms a single GOE sequence.

We can follow this transition better by calculating the spectral rigidity which is, for this system, given by

$$\Delta_3(L) = 2\Delta_{3,\mathrm{GOE}}\left(\frac{fL}{2}\right) + \Delta_{3,\mathrm{GOE}}[(1-f)L]. \tag{15}$$

In figure 2, the results of the calculation for $f = 1$, 0.9, 0.667, 0.333, 0.1 and 0 are given by the curves labelled by A, B, C, D, E and F, respectively, while the curve P corresponds to a regular spectrum. The figure clearly shows that $\Delta_3(L)$ increases as symmetry breaking increases, and thus evolves towards the regular shape until f reaches the value of 2/3. Only then $\Delta_3(L)$ starts a route leading to the GOE curve.

We believe that the first model applies well to the cases when the Hamiltonian can be represented as a superposition of two terms, one conserving the symmetry and the other violating it. In nuclei, the isospin symmetry is violated by switching on the Coulomb force. Indeed, the Guhr–Weidenmüller model [11] works well. However, we cannot be sure that this model will work in cases when the symmetry-breaking term of the Hamiltonian cannot be explicitly isolated. An example is the experiment with accoustic blocks [14] where symmetry is violated by removing sections of the blocks. We shall see in the next section that the proposed scenario, formulated by (12), is capable of reproducing the observed influence of the symmetry breaking on the resonance spectra.

3. Comparison with acoustic resonance spectra

In this section, we show that the proposed application of the Berry–Robnik theory to symmetry breaking leads to a satisfactory description of the acoustic resonance spectra of monocrystalline quartz blocks, measured by Ellegaard *et al* [14]. Crystalline quartz exhibits D_3 point-group symmetry about the crystal's Z (optical) axis and three two-fold rotation symmetries about the three X (piezoelectric) axes; the latter three axes lie in a plane orthogonal to the Z axis and sustain angles of 120° with respect to each other. Ellegaard *et al* used rectangular blocks of dimensions $14 \times 25 \times 40$ mm^3, cut in such a way that all symmetries are fully broken except a two-fold 'flip' symmetry about one of the three X axes. A gradual breaking of this symmetry is then achieved by removing successively bigger octants of a sphere from one corner, thereby creating a three-dimensional Sinai billiard. The acoustic spectrum is measured for eight radii of the removed octant, providing data of high statistical significance for the study of the gradual breaking of a point-group symmetry.

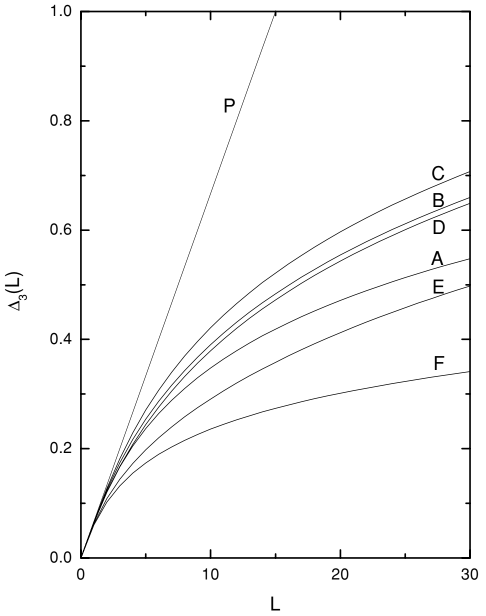

Figure 2. The spectral rigidity $\Delta_3(L)$ for a chaotic system undergoing a gradual breaking of a symmetry according to the scenario proposed in the paper. The results of the calculation when the fraction of states remains nearly symmetry invariant $f = 1$, 0.9, 0.667, 0.333, 0.1 and 0 are given by the curves labelled by A, B, C, D, E and F, respectively, while the curve P corresponds to a regular spectrum.

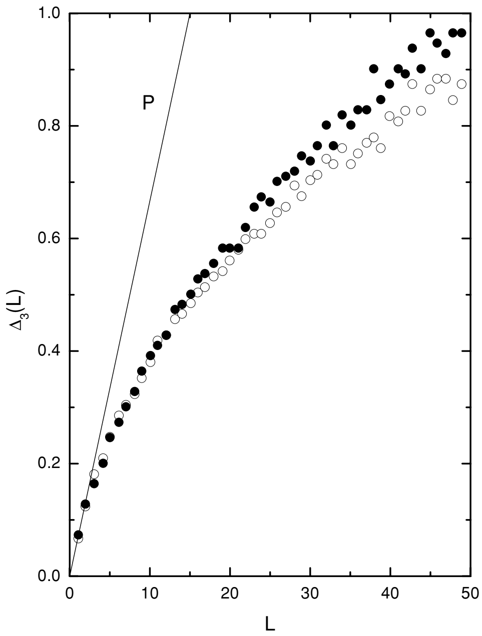

Figure 3. The spectral rigidity $\Delta_3(L)$ measured in [14] for the quartz block when the symmetry is present (open circles), and when it is violated by removing a tiny octant of radius $r = 0.5$ mm (closed circles). The straight line labelled P is for an integrable system.

Figure 3 shows the spectral rigidity for the cases when the symmetry is present (open circles), and when it is violated by removing a tiny octant of radius $r = 0.5$ mm (closed circles). The straight line labelled P is for an integrable system. We note that the data for the case of tiny symmetry violation are systematically higher than those for the case of conserved symmetry. This agrees with the prediction of the second scenario proposed above, in which the system that undergoes a gradual symmetry breaking looks as if it has become more regular in the early stages of the transition.

Ellegaard *et al* [14] concluded from the rise of Δ_3 over the theoretical prediction for a superposition of two GOE sequences that the quartz block with conserved flip symmetry has much in common with a pseudo-integrable (PI) system. They supported this conclusion by means of other measurement. PI systems are non-integrable, yet non-chaotic systems. Two-dimensional PI systems, exemplified by particles moving in a planar polygonal enclosure with rational angles [5], share with the integrable system the motion of trajectories in the phase space which is restricted to two-dimensional compact surfaces. But these invariant surfaces are multi-connected and not tori as in the case of integrable systems. Numerical studies of the $\pi/3$ rhombus billiard [6, 22, 23] show that the spectral fluctuation properties are intermediate between those of a Poisson ensemble and a GOE. Bogomolny *et al* [23] studied the level statistics of PI systems using a short-range Dyson model. This is a modified version of Dyson's stochastic Coulomb-gas model [15], in which the interaction between particles is restricted only to the nearest neighbours. The NNS distribution and the two-point correlation function obtained in [23] are, respectively, given by the so-called semi-Poissonian statistics:

$$P_{\mathrm{SP}}(s) = 4s\,\mathrm{e}^{-2s} \qquad \text{and} \qquad R_{2,\mathrm{SP}}(s) = 1 - \mathrm{e}^{-4s}. \tag{16}$$

The same functional form was used earlier by Date *et al* [7] to explain the intermediate spacing distribution for a rectangular billiard with a perpendicular flux line. It is known, however, that the spectral statistics of PI systems are 'non-universal with universal trend' [24]. Jain and collaborators [25, 26] generalized the short-range Coulomb-gas model by taking into account both the nearest- and next-to-nearest-neighbour interactions. They have shown that there is a parameter β_{SI}, which relates the strengths of these types of interactions in the cases when a complete bound-state spectrum of the Coulomb gas is available. Among other things, they obtained the following expression for the nth neighbour level-spacing distribution ($p(n+1, s)$ in Mehta's notation [15]):

$$P^{(\beta_{\mathrm{SI}})}(n, s) = \frac{(\beta_{\mathrm{SI}} + 1)^{n(\beta_{\mathrm{SI}}+1)}}{\Gamma[n\,(\beta_{\mathrm{SI}} + 1)]} s^{n(\beta_{\mathrm{SI}}+1)-1} \mathrm{e}^{-(\beta_{\mathrm{SI}}+1)s}. \tag{17}$$

Therefore, the parameter β_{SI} defines the power of level repulsion. Their results for $\beta_{\mathrm{SI}} = 1$ yield the semi-Poissonian statistics. They also obtained closed-form expressions for the two-point correlation functions $R_2^{\beta_{\mathrm{SI}}}(s)$ for other integral values of β_{SI}—see specifically equations (63), (65), (66), (68) and figure 1 of [26].

The spectral rigidity for the PI systems is expressed in terms of the two-point correlation function [1]. Thus, using the results of [26], we obtain the following expressions for the spectral regidity $\Delta_3^{(\beta_{\mathrm{SI}})}(L)$ of the spectra for the level-repulsion parameter $\beta_{\mathrm{SI}} = 1$–4:

$$\Delta_3^{(1)}(L) = \frac{L}{30} + \frac{1}{16} - \frac{1}{16L} + \frac{1}{32L^2} - \frac{3}{512L^4} + \mathrm{e}^{-4L}\left(\frac{1}{64L^2} + \frac{3}{128L^3} + \frac{3}{512L^4}\right) \tag{18}$$

$$\Delta_3^{(2)}(L) = \frac{L}{45} + \frac{2}{27} - \frac{4}{81L} + \frac{8}{729L^2} + \frac{8}{6561L^4} + \frac{4\mathrm{e}^{-4L}}{6561L^4}\left[(9L^2 - 2)\cos\left(\frac{3\sqrt{3}}{2}L\right) - \sqrt{3}(9L^2 + 12L + 2)\sin\left(\frac{3\sqrt{3}}{2}L\right)\right] \tag{19}$$

$$\Delta_3^{(3)}(L) = \frac{L}{60} + \frac{5}{64} - \frac{5}{128L} + \frac{1}{512L^2} + \frac{45}{32\,768L^4} + \frac{\mathrm{e}^{-8L}}{32\,768L^4}(32L^2 + 24L + 3) - \frac{\mathrm{e}^{-4L}}{2048L^4}[3(4L + 1)\cos(4L) + 4L(4L + 3)\sin(4L)] \tag{20}$$

and

$$\Delta_3^{(4)}(L) = \frac{L}{15} + \frac{2}{15L^4}\left[I\left(\frac{5\pi}{2}, L\right) + I\left(\frac{5\pi}{4}, L\right)\right] \tag{21}$$

where

$$I(a, x) = \mathrm{Re}\int_0^x (x - r)^3(2x^2 - 9xr - 3r^2)\mathrm{e}^{-5(1-\cos a - \mathrm{i}\sin a)r}\,\mathrm{d}r. \tag{22}$$

We then calculate the corresponding spectral rigidities of an independent superposition of two level sequences of equal densities, which is equal to $2\Delta_3^{\beta_{\mathrm{PI}}}(L/2)$ according to Seligman and Verbaarschot [20]. Figure 4 shows the result of calculation with the spectral rigidity of the complete quartz cube [14], where the two-fold flip symmetry is conserved. We see from the figure that the data agree with the curve for $\beta_{\mathrm{PI}} = 1$ when $L \lesssim 5$. This suggests that the

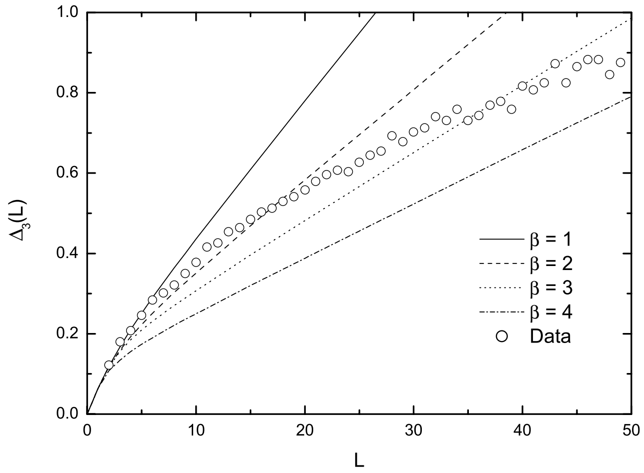

Figure 4. The spectral rigidity $\Delta_3(L)$ measured in [14] for the quartz block when the symmetry is present (open circles) compared to the predictions of the short-range Dyson model [25, 26] for different values of the level-repulsion parameter β.

semi-Poissonian statistics is reasonable for the spectrum under investigation as far as relatively small spacings are concerned. As we increase L, the experimental data seem to be requiring larger values of $\beta_{\rm PI}$. In fact equations (18–21) suggest that

$$\Delta_3^{\beta_{\rm PI}}(L) \sim \frac{1}{15(\beta_{\rm PI}+1)L} \tag{23}$$

for large values of L. However, the data do not show any tendency to saturate to a staight line, but rather have a negative gradient everywhere. We therefore follow several authors, e.g. [6], and represent the Δ_3 statistic of the PI system by the Seligman–Verbaarschot [20] formula (9), obtained for a superposition of Poisson and GOE level sequences,

$$\Delta_{3,\rm PI}(L) = q\frac{L}{15} + \Delta_{3,\rm GOE}[(1-q)L] \tag{24}$$

where q is a fitting parameter. Biswas and Jain [6] found q to be equal to 0.2 for the $\pi/3$ rhombus billiard. We do this by fitting the same statistic for the spectrum of the complete quartz cube to $2\Delta_{3,\rm PI}(L/2)$. We obtain $q = 0.082$.

We now apply the second scenario, which has been proposed in the previous section, to the symmetry breaking in a PI system having a symmetry represented by a quantum number that takes one of two possible values. When the symmetry is conserved, the spectrum is an independent superposition of two sequences corresponding to the two representations of the symmetry. The NNS distribution is accordingly represented by (2) with $n = 2$ and the Δ_3 statistic by (9). The functions $P_i(x_i)$ and $\Delta_{3,i}(L)$ are given by (16) and (24), respectively. Now we again assume that as the symmetry violation starts, the eigenfunctions of the system are divided into two classes. The first constitutes a fraction f of the eigenfunctions in which the perturbation due to symmetry breaking can be ignored. The levels of these eigenstates are divided into two independent sequences exactly as before switching the symmetry breaking on. The second is the class of strongly perturbed functions that have completely lost the symmetry. The latter class will here be modelled by a GOE. Therefore, the total spectrum

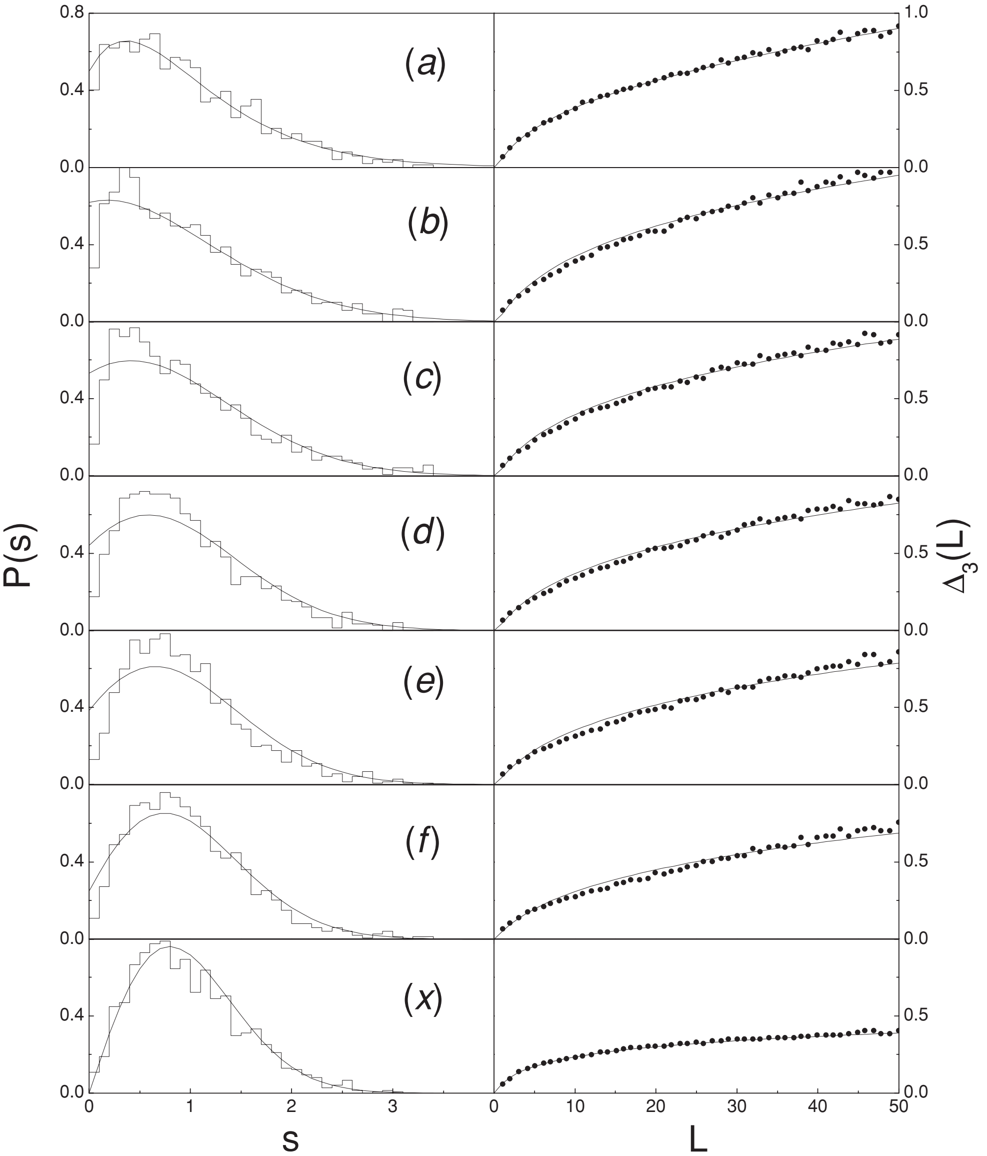

Figure 5. The NNS distributions $P(s)$ and spectral rigidities $\Delta_3(L)$ for different radii of the octant removed from a quartz block: (*a*) $r = 0$, the flip symmetry is fully conserved, (*b*) $r = 0.5$ mm, (*c*) $r = 0.8$ mm, (*d*) $r = 1.1$ mm, (*e*) $r = 1.4$ mm, (*f*) $r = 1.7$ mm, (*x*) the block with the huge defocusing structure. The data are from [14], while the curves are the theoretical curves using equations (25) and (26). Values of the parameter f that measures fractional densities of states that keep the symmetry are given in table 1.

will be composed of three independent sequences, two with PI statistics, each having a fractional density of $f/2$, and one with GOE statistics and fractional density $1 - f$. During the symmetry-breaking transition, the NNS distribution is given by

Table 1. Best fit values for the fraction f of states not affected as a result of violating the point-group symmetry of a quartz rectangular block by removing spherical octants of various radii r.

r	0	0.5 mm	0.8 mm	1.1 mm	1.4 mm	1.7 mm	Huge defocussing
f	1	0.49	0.36	0.29	0.23	0.17	0

$$P(s) = \frac{1}{2} f^2(2f^2s^2 + 4fs + 1)\mathrm{e}^{-2fs}\,\mathrm{erfc}\left[\frac{\sqrt{\pi}}{2}(1-f)s\right] + \frac{1}{8}(1-f)(2+fs)\mathrm{e}^{-\frac{\pi}{4}(1-f)^2s^2-2fs} \times[\pi f s^2(1-f)^2 + 2s\{(\pi+4)f^2 - 2\pi f + \pi\} + 8f] \tag{25}$$

which is obtained by substituting equations (3) and (16) into (2). The Δ_3 statistic is given by

$$\Delta_3(L) = 2\Delta_{3,\mathrm{PI}}\left(\frac{fL}{2}\right) + \Delta_{3,\mathrm{GOE}}[(1-f)L]. \tag{26}$$

We now compare our model with the acoustic resonance spectra measured by Ellegaard *et al* [14]. We start by estimating the parameter q in the expression (17) for the Δ_3 statistic of the PI system. We then compare equation (26) with the spectral rigidities of the other spectra considering f as a free parameter. The best-fit values of f for spectra corresponding to different radii of the removed octants are given in table 1. We use these values to calculate the NNS distributions. The results of the calculation are compared with the experimental data in figure 5. We see that the proposed model presents a satisfactory description of the whole transition by varying a single parameter. The only disagreement is between the calculated and measured values of $P(s)$ at small values of s, as we have already expected. The symmetry breaking decreases the probability of finding degenerate levels sharply, leading to the observed dip at small s in the spacing distribution [14]. This dip is followed by an overshoot to restore normalization. The width of this dip, which is about $1/10$ of the mean level spacing, provides an estimate for the minimum level-splitting below which two levels can be regarded as approximately degenerate.

4. Summary and conclusion

Conventional models for symmetry breaking in chaotic systems assume that the non-diagonal matrix elements of the symmetry-breaking perturbation are statistically equivalent. They predict a monotonic transition from a mixed statistics to the GOE as one switches on and increases the symmetry-breaking perturbation. This is not the behaviour of the acoustic resonance spectra of quartz measured by Ellegaard *et al*. We propose another scenario for symmetry breaking in which the levels are approximately separated into two groups, one with and one without symmetry. This happens when the expansion of any of the wavefunctions in terms of the eigenfunctions of the symmetry operator is either dominated by a single term or composed of statistically equivalent contributions from all its components. Considering a system in which a symmetry that has two representations is conserved, we represent the spectrum as a superposition of two independent sequences, one for each symmetry representation. Now, as we switch on the symmetry-breaking perturbation, we remove a number of levels from these two level sequences into a third sequence. The latter corresponds to the strongly perturbed states in which the symmetry has virtually disappeared. Because we have increased the number of spectral partitions, the spectrum will look as if it became more regular by the small violation of the symmetry. The apparent regularity will continue until the three sequences become equally populated. The further increase of symmetry breaking will reverse the evolution of the shapes of the spectral statistics towards the GOE shapes. Namely, this is the behaviour found by inspection of the acoustic resonance spectra of quartz measured by Ellegaard *et al*. We see, for instance, from figure 5 that it is possible to describe the evolution of the NNS distribution and the spectral rigidity during the transition from a fully conserved to a fully violated symmetry by varying a single parameter, measuring the fractional density of states which are practically unaffected by symmetry violation.

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